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Black hole dynamics from thermodynamics in anti-de Sitter space

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Abstract

We work on the relation between the local thermodynamic instability and the dynamical instability of large black holes in four-dimensional anti-de Sitter space proposed by Gubser and Mitra. We find that the chance is higher than expected that black holes become dynamically unstable when they lose the local thermodynamic stability.

1 Introduction

Black holes are very interesting objects from their causal structures in general relativity to their quantum mechanical properties. To figure out their physical relevance, we need to answer if the complete gravitational collapse of a body results in a black hole rather than a naked singularity. The conjecture [1] that nature censors naked singularity was proposed in this respect. One of motivation of this is from the fact that black holes in 4-dimensional asymptotic Minkowski space are stable: linear perturbations around black hole solutions do not give any evolution.

However, it was found in [2, 3] that black strings and p-branes are unstable. The basic idea of the Gregory-Laflamme instability is that whatever has the biggest entropy is favored. Since a black string has a different topology of horizon as that of a black hole and entropy is proportional to the area of horizon, array of black holes has bigger entropy than a uncompactified black string when they have the same mass [4]. The instability of black strings was shown [2, 3] by doing perturbation theory. It is a very interesting question to see what would happen during the transition between them. It has been argued that violation of cosmic censorship does occur during this process. Recently it has been suggested that a black string settles down to a new static black string solution which is not translationally invariant along the string [5].

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Entropy argument used above was revisited by Gubser and Mitra to propose that a black brane becomes dynamically unstable when it is locally thermodynamically unstable [6, 7]. Local thermodynamic stability is defined as having an entropy which is concave down as a function of the mass and the conserved charges [8]. This conjecture was made from the perspective of AdS/CFT correspondence [9, 10, 11], which identifies two low energy excitations, both of which are decoupled from supergravity in flat space, in two low energy descriptions of superstring theory [12]. Some unstable fluctuation modes may be excited when there is a thermodynamic instability in the field theory and according to AdS/CFT the same thing would happen in AdS [7]. A semi-classical proof of above conjecture using the Euclidean path integral approach to quantum gravity was given in [13].

The motivation of Gubser-Mitra (GM) conjecture is that Lorentzian time evolution should proceed so as to increase the entropy. In this paper, it is found that there is some ambiguity in this argument. Sometimes the evolution goes in the direction of decreasing entropy: In the case that the metric fluctuations are suppressed, any perturbation for all equal charges of AdS_4-RN solution becomes dynamically unstable when the system loses the thermodynamic stability and some perturbations decrease entropy in this case. On the other hand, when only the metric fluctuations are turned on, there is no dynamical instability even though the system is thermodynamically unstable. This stability can be explained by the fact that entropy would be decreasing if the perturbation is unstable. We discuss the former in section 2 and the latter in section 3. We try to use this contradiction to justify their conjecture in section 4.

2 Gubser-Mitra analysis and its generalization

2.1 AdS_4-RN black hole

An electrically charged black hole in the asymptotically AdS_4 was found in [14]. Starting from $\mathcal{N}=8$ supergravity in 4-dimensions, they gauged the rigid SO(8) symmetry of 28 gauge boson [15] and the potential induced from this gauging makes AdS_4 a vacuum solution of this theory. The AdS_4 black hole solution is made by focusing on $U(1)^4$ Cartan subgroup of SO(8), which is believed to be a consistent truncation. Only three scalar fields of 70 scalar fields in the original theory are kept by working in symmetric gauge. The Lagrangian of this truncated theory is,

$$\mathcal{L} = \frac{\sqrt{g}}{2k^2}\left[R - \sum_{i=1}^{3}\left(\frac{1}{2}(\partial\phi_i)^2 - \frac{2}{L^2}\cosh\phi_i\right) - 2\sum_{A=1}^{4} e^{\alpha^i_A\phi_i}(F^{(A)}_{\mu\nu})^2\right] \tag{1}$$

$$\text{where}\quad \alpha^i_A = \begin{pmatrix} 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}.$$

The metric signature is $(-+++)$ and $G_4 = \frac{1}{4}$. The electrically charged solutions are

$$
\begin{aligned}
ds^2 &= -\frac{F}{\sqrt{H}}dt^2 + \frac{\sqrt{H}}{F}dz^2 + \sqrt{H}z^2 d\Omega^2 \\
e^{2\phi_1} &= \frac{h_1 h_2}{h_3 h_4} \quad e^{2\phi_2} = \frac{h_1 h_3}{h_2 h_4} \quad e^{2\phi_3} = \frac{h_1 h_4}{h_2 h_3} \\
F_{0z}^{(A)} &= \pm \frac{1}{\sqrt{8}h_A^2}\frac{Q_A}{z^2} \\
H &= \prod_{A=1}^{4} h_A \quad F = 1 - \frac{\mu}{z} + \frac{z^2}{L^2}H \quad h_A = 1 + \frac{q_A}{z} \\
Q_A &= \mu \cosh\beta_A \sinh\beta_A \quad q_A = \mu \sinh^2\beta_A
\end{aligned}
\tag{2}
$$

where the quantities Q_A are the physical conserved charges. The mass is [16]

$$
M = \frac{\mu}{2} + \frac{1}{4}\sum_{A=1}^{4} q_A, \tag{3}
$$

and the entropy is

$$
S = \pi z_H^2 \sqrt{H(z_H)} \tag{4}
$$

where z_H is the largest root of $F(z_H) = 0$. It is possible to express M directly in terms of the entropy and the physical charges in the large black hole limit, $M \gg L$, as [7]

$$
M = \frac{1}{2\pi^{\frac{3}{2}}L^2\sqrt{S}}\left[\prod_{A=1}^{4}(S^2 + \pi^2 L^2 Q_A^2)\right]^{\frac{1}{4}}. \tag{5}
$$

We are going to study in the case where all four charges are equal, $q_A = q$. In this case the solution can be written in term of a new radial variable, $r = z + q$, and it becomes

$$
\begin{aligned}
ds^2 &= -f dt^2 + \frac{dr^2}{f} + r^2 d\Omega^2 \\
F_{0r} &= \frac{Q}{\sqrt{8}r^2} \\
f &= 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{L^2}.
\end{aligned}
\tag{6}
$$

It is known [17] that the consistent S^7 truncation of 11-dimensional supergravity is equivalent to $\mathcal{N} = 8$ 4-dimensional gauged supergravity. Also the equivalence of large R-charged black holes in D=4, D=5 and D=7 with spinning near-extreme M2, D3 and M5 branes are respectively demonstrated in [16]. It is important to check that our black holes can be embedded to higher dimensional black objects because GM conjecture requests the non-compact translational symmetry. Any instability found in the large black hole limit, $M \gg L$ in our case implies the instability of M2-brane.

2.2 Thermodynamic instability and adiabatic evolution

Local thermodynamic stability is defined as having an entropy which is concave down as a function of the extensive variables. This means that the Hessian matrix,

$$H^S_{M,Q_A} \equiv \begin{pmatrix} \frac{\partial^2 S}{\partial M^2} & \frac{\partial^2 S}{\partial M \partial Q_B} \\ \frac{\partial^2 S}{\partial Q_A \partial M} & \frac{\partial^2 S}{\partial Q_A \partial Q_B} \end{pmatrix} \tag{7}$$

has no positive eigenvalues. It is straightforward to express H^S_{M,Q_A} in terms of derivatives of $M(S, Q_A)$. From this definition, if we introduce the dimensionless variable $\chi = \frac{Q}{M^{\frac{2}{3}} L^{\frac{1}{3}}}$ for all equal charges, $Q_A = Q$, the thermodynamic instability is present when $\chi > 1$ [7].

GM conjecture was motivated from the intuition that Lorentzian time evolution should proceed so as to increase the entropy. We can see that the most positive eigenvetor[1] of Hessian increases entropy most when entropy is at its extremum.

$$S(M + \delta M, Q_A + \delta Q_A) = S(M, Q_A) + (\delta M, \delta Q_A) H^S_{M,Q_A} \begin{pmatrix} \delta M \\ \delta Q_B \end{pmatrix} \tag{8}$$

Even though entropy is not at its extremum, we can forget about the first derivative parts by energy and charge conservation in microcanonical ensemble. With this they found that in the positive eigenvector direction of Hessian for all equal charges, the dynamical instability coincides with the thermodynamic instability. It will be interesting to see what would happen in other directions.

Gubser and Mitra analyzed the linear perturbation in which fluctuations of the metric are suppressed. The most unstable eigenvector is $(\delta M, \delta Q_A) = (0, 1, 1, -1, -1)$ for all equal charges. The condition that the metric decouples at linear order is that $Q_A \cdot \delta Q_A = 0$. In this case δT_{ab} vanishes at linear order and we can also see from (2) the metric does not change at linear order. It is not difficult to make the linear perturbation equations in this decoupling case beyond the eigenvector direction. Our original motivation was to see two things: first, even though the system loses the thermodynamic stability, it would not be dynamically unstable if we perturb the system in the way of decreasing its entropy. Second, because the eigenvector direction increases entropy most, it would be the fastest way of increasing entropy.

We found that both of above reasoning are incorrect. The general perturbation in which the metric decouples for all equal charges is that

$$\delta Q_A = (1, a, b, -a - b - 1)\delta Q \tag{9}$$

where a, b can be any real numbers. From (2), we can make an ansatz about a relevant perturbation

$$\begin{aligned} \delta\phi_i &= (1 + a, 1 + b, -a - b)\frac{\delta\phi}{2} \\ \delta F^{(A)} &= (1, a, b, -a - b - 1)\delta F. \end{aligned} \tag{10}$$

This ansatz relating three scalar fields to one scalar field and four U(1) fields to the other U(1) field should be checked if it is consistent with equations of motion and

[1] The positive eigenvector here means an eigenvector with a positive eigenvalue.

it turns out to be consistent. The linear perturbation for each ϕ_i is the same up to overall factor

$$\left[\nabla_\mu \nabla^\mu + \frac{2}{L^2} - 8F_{\mu\nu}^2\right] \delta\phi - 16F^{\mu\nu}\delta F_{\mu\nu} = 0 \tag{11}$$

and the linear perturbation for each $F_{\mu\nu}^{(A)}$ is also the same up to overall factor

$$d\delta F = 0 \qquad d * \delta F + d\delta\phi \wedge *F = 0. \tag{12}$$

Here F is the background field strength in (6): it is the same for four $F^{(A)}$. It is remarkable that all directions have the same perturbation equation. The case of $a\!=\!1$, $b\!=\!-1$ is the unstable eigenvector and we can see that (11) and (12) are exactly what Gubser and Mitra found [6]. From their analysis, we can conclude that all perturbations suppressing metric fluctuation at linear order have dynamical instabilities when the system is thermodynamically unstable. If χ is slightly greater than 1, only small neighbors around the eigenvector direction increase entropy and most of perturbations (9) decrease entropy. However, the system is dynamically unstable in all cases.

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3 Stability from the metric perturbation analysis

3.1 Metric perturbation equation

In the previous section, we observed that there is a dynamical instability even though the evolution does not increase entropy. It would be very interesting to see what would happen in the case that metric is also involved. This is very difficult to do and it has not been cleared up what would be the difference between the linear perturbation equation for the most unstable direction which is the eigenvector of Hessian and those for other directions. For example, suppose $(\delta M, \delta Q_A)$ is a positive eigenvector of Hessian for some value (M, Q_A) and we have a perturbation equation for this. It is hard to see that this perturbation equation would be different from the perturbation of $(\delta M/2, \delta Q_A)$, which is not the eigenvector of Hessian.

In this section, we analyze a simple case: three scalar fields and four U(1) fields are suppressed. This perturbation is in the direction of $\delta M\!\neq\!0$, $\delta Q_A\!=\!0$ for all equal charges. From (5) and (8) we can see that entropy is decreasing in this perturbation. Varying (1) yields the equations of motion

$$\nabla_a \nabla^a \phi_i + \frac{2}{L^2}\sinh\phi_i - 2\sum_{A=1}^{4} \alpha^i_A e^{\alpha^j_A \phi_j} (F^{(A)}_{ab})^2 = 0 \tag{13.a}$$

$$\partial_a(\sqrt{g} e^{\alpha^i_A \phi_i}(F^{(A)\,ab})) = 0 \tag{13.b}$$

$$R_{ab} = -\sum_{i=1}^{3}\left\{\frac{1}{L^2}\cosh\phi_i g_{ab} + \frac{1}{2}\partial_a\phi_i\partial_b\phi_i\right\} + 4\sum_{A=1}^{4}\left\{F^{(A)}_{ac}F^{(A)\,c}_{\ \ b} - \frac{1}{4}g_{ab}(F^{(A)})^2\right\}. \tag{13.c}$$

We expect a relevant perturbation is

$$\delta\phi_i \;=\; 0$$

$$
\begin{aligned}
\delta F^{(A)}_{ab} &= 0 \qquad (14)\\
\delta g_{ab} &= \gamma_{ab}.
\end{aligned}
$$

We need to check if above ansatz is consistent with equations of motion and it is so if

$$\gamma = 0 \qquad \gamma^t_{\ t} + \gamma^r_{\ r} = 0 \tag{15}$$

where $\gamma = \gamma^a_{\ a} = g^{ab}\gamma_{ba}$. The linear perturbation equations from (13.a) and (13.b) are automatically satisfied for all equal charges . Now we need to check if (15) is consistent with (13.c). The linear perturbation from (13.c) is

$$
\begin{aligned}
& -\frac{1}{2}\nabla_a\nabla_c\gamma - \frac{1}{2}\nabla^b\nabla_b\gamma_{ac} + \nabla^b\nabla_{(c}\gamma_{a)b} \\
= \ & \Lambda\gamma_{ac} + 16\left(-F_a^{\ b}F_c^{\ d}\gamma_{bd} - \frac{1}{4}\gamma_{ac}F^2 + \frac{1}{2}g_{ac}F^{bd}F^e_{\ d}\gamma_{be}\right) \qquad (16)
\end{aligned}
$$

where we use the totally symmetric notation for ( ), $\Lambda = -\frac{3}{L^2}$ and $F^2 = F^{ab}F_{ab}$. Four U(1) fields become the same. It is a famous story from electromagnetism that if there is a source term, a simple gauge choice is not easy. However, if we take the trace of (16), it becomes

$$-\frac{1}{2}\nabla_a\nabla^a\gamma = \Lambda\gamma - 16\gamma F^2. \tag{17}$$

We used the condition, $\gamma^t_{\ t} = -\gamma^r_{\ r}$ for this. Because (17) is a homogeneous equation for γ, we can choose a transverse traceless gauge whereby

$$
\begin{aligned}
\nabla^a\gamma_{ab} &= 0\\
\gamma &= 0. \qquad (18)
\end{aligned}
$$

See [18] for a detail about this gauge choice. Finally we get the perturbation equation for the metric from (16) following [18].

$$(\nabla^b\nabla_b + 2\Lambda - 8F^2)\gamma_{ac} - (R_c^{\ d}\gamma_{ad} + R_a^{\ d}\gamma_{cd}) - (2R^{b\ \ d}_{\ ac} + 32F_a^{\ b}F_c^{\ d})\gamma_{bd} = 0 \tag{19}$$

It is the even wave in the canonical form [19] which is relevant to our case:

$$\gamma_{ab} = \begin{pmatrix} \tilde{\gamma}_{tt} & \tilde{\gamma}_{tr} & 0 & 0\\ \tilde{\gamma}_{rt} & \tilde{\gamma}_{rr} & 0 & 0\\ 0 & 0 & r^2k & 0\\ 0 & 0 & 0 & r^2k\sin^2\theta \end{pmatrix} e^{iwt}P_l(\cos\theta). \tag{20}$$

It can be easily checked that in this form, $\gamma^t_{\ t}$ and $-\gamma^r_{\ r}$ have the same equation in (19) and this proves that our ansatz (14)-(15) is completely consistent with equations of motion. This equality between $\gamma^t_{\ t}$ and $-\gamma^r_{\ r}$ is expected from the δM perturbation of the metric in (6). The equations for γ_{tt} and γ_{tr} are coupled:

$$
\begin{aligned}
&\left\{-\frac{\partial_t^2}{f} + f\partial_r^2 + \left(2\frac{f}{r} - f'\right)\partial_r - 2\frac{f'}{r} + \frac{\partial_\theta\sin\theta\partial_\theta}{r^2\sin\theta}\right\}\gamma_{tt} + 2f'\partial_t\gamma_{tr} = 0 \qquad (21)\\
&\left\{-\frac{\partial_t^2}{f} + f\partial_r^2 + \left(2\frac{f}{r} + f'\right)\partial_r - \frac{(f')^2}{f} - f'' + \frac{\partial_\theta\sin\theta\partial_\theta}{r^2\sin\theta}\right\}\gamma_{tr} + \frac{2f'\partial_t}{f^2}\gamma_{tt} = 0
\end{aligned}
$$

Here f is defined in (6) and $f' = \partial_r f$. Using the form (20) we can make the forth order ordinary differential equation for $\tilde{\gamma}_{tr}$, which is what we are going to study by numerics.

3.2 Numerical analysis

To carry out a numerical study of (21), we can cast the equation in terms of a dimensionless radial variable u, a dimensionless charge parameter χ, a dimentionless mass parameter σ, and a dimensionless frequency $\tilde{w}$ introduced in [6]:

$$u = \frac{r}{M^{\frac{1}{3}} L^{\frac{2}{3}}} \quad \chi = \frac{Q}{M^{\frac{2}{3}} L^{\frac{1}{3}}} \quad \sigma = \left(\frac{L}{M}\right)^{\frac{2}{3}} \quad \tilde{w} = \frac{w L^{\frac{4}{3}}}{M^{\frac{1}{3}}} \tag{22}$$

Then we combine two equations in (21)

$$\begin{aligned} &\left[\left\{\frac{\tilde{w}^2}{\tilde{f}} + \tilde{f}\partial_u^2 + \left(2\frac{\tilde{f}}{u} - \tilde{f}'\right)\partial_u - 2\frac{\tilde{f}'}{u} - \frac{\sigma l(l+1)}{u^2}\right\}\frac{\tilde{f}^2}{2\tilde{f}'}\right. \\ \times \quad &\left.\left\{\frac{\tilde{w}^2}{\tilde{f}} + \tilde{f}\partial_u^2 + \left(2\frac{\tilde{f}}{u} + \tilde{f}'\right)\partial_u - \frac{(\tilde{f}')^2}{\tilde{f}} - \tilde{f}'' - \frac{\sigma l(l+1)}{u^2}\right\} + 2\tilde{f}'\tilde{w}^2\right]\tilde{\gamma}_{tr} = 0 \\ &\tilde{f} = \sigma - \frac{2}{u} + \frac{\chi^2}{u^2} + u^2. \end{aligned} \tag{23}$$

Now we need to specify the boundary condition for $\tilde{\gamma}_{rt}$. We want to place a initial data surface touching the horizon at one end and ending on the boundary of AdS at the other end. Because AdS does not have a Cauchy surface [20], the domain of dependence of this initial data lies inside of Cauchy horizon of AdS. To define 'small' for the perturbation at the horizon, we can use non-singular coordinates, Kruskal coordinates [21]. Dropping the S^2 piece in (6) and introducing a tortoise coordinate r_* and Kruskal coordinates (T, R) according to

$$\begin{aligned} \frac{dr_*}{dr} &= \frac{1}{f} \\ e^{\frac{1}{2}f'(r_H)(\pm t + r_*)} &= \pm T + R. \end{aligned} \tag{24}$$

The near-horizon metric is regular

$$ds^2 \approx \frac{4}{e^{f'(r_H) r_H} f'(r_H)}(-dT^2 + dR^2). \tag{25}$$

We can express the Kruskal components γ'_{ab} in terms of the original components γ_{ab}

$$\begin{aligned} \gamma'_{tt} &= \frac{4}{f'(r_H)(-T^2+R^2)^2}\left[R^2\gamma_{tt} - 2fRT\gamma_{tr} + f^2T^2\gamma_{rr}\right] \\ \gamma'_{tr} &= \frac{4}{f'(r_H)(-T^2+R^2)^2}\left[-TR\gamma_{tt} + 2f(T^2+R^2)\gamma_{tr} - f^2TR\gamma_{rr}\right] \\ \gamma'_{rr} &= \frac{4}{f'(r_H)(-T^2+R^2)^2}\left[T^2\gamma_{tt} - 2fRT\gamma_{tr} + f^2R^2\gamma_{rr}\right] \end{aligned} \tag{26}$$

all of which should be finite as $r \to r_H$ on our initial data surface. To avoid the issue of mode superposition and a better physical sense in which black holes would form in a collapse situation, we would require a surface ending on a future horizon [22]. When we approach the future horizon from outside of a black hole region, $R = T + O(r - r_H)$.

This implies that normalizable wavefunctions γ_{tr} must be $O(r-r_H)$ as we approach the event horizon. We also want it falling off like $\frac{1}{r^2}$ near the boundary of AdS_4. Using Maple, we solved (23) numerically. We did not find any unstable mode. At $\sigma=0$, thermodynamic stability is lost at $\chi=1$. The smallest $\tilde{w}^2$ in this case is $\tilde{w}^2=2$. There is no normalizable wavefunction with negative $\tilde{w}^2$. Negative mode is found at $\chi=3.7$, which lies in naked singularity regions and therefore is not relevant. We can conclude that in the perturbation, $\delta M\neq 0$, $\delta Q_A=0$ there is no dynamical instability, which makes sense because this perturbation would decrease entropy if it were unstable.

4 Conclusions and Perspectives

We have been working on Gubser-Mitra conjecture, which relates thermodynamics to dynamics in black holes. We have a seemingly contradictory result: When the metric fluctuations are suppressed at linear order, all perturbations are dynamically unstable if black holes are thermodynamically unstable and some of evolutions decrease entropy. However, when only the metric fluctuations are turned on, they are dynamically stable. Entropy is not decreasing in this case. This result might weaken the motivation of Gubser-Mitra conjecture, which claims that Lorentzian time evolution should go so as to increase entropy. However, this result might be used to strengthen the validity of their conjecture.

Suppose we have a hypersurface defined by $S=S(M,Q_A)$. Each pertuabation $(\delta M,\delta Q_A)$ corresponds a tangent vector originated from $p=(M_0,Q_{0A})$ up to a normalization factor in a tangent space T_pS of the hypersurface at p [2]. If $S(p)$ is no longer local maximum of S, there is a positive eigenvector of Hessian (7) in the tangent space, which means the local thermodynamic instability. If Lorentzian time evolution should increase entropy, it would be very hard for the system to be dynamically unstable as soon as it becomes thermodynamically unstable, because the system should be perturbed exactly in the direction of the positive eigenvector of Hessian. As we found, if perturbations around the eigenvector direction are also dynamically unstable as soon as the system becomes thermodynamically unstable, the chance for the system to be dynamically unstable when it becomes thermodynamically unstable is very high. To be more precise, we can separate the tangent space into a stable region in which a perturbation vector $(\delta M,\delta Q_A)$ giving no evolution is ending, and a unstable region in which a perturbation vector giving a evolution is ending. As we found, perturbation vectors $(\delta M,\delta Q_A)=(0,1,a,b,-1-a-b)$ in section 2 are in the unstable region and perturbation vectors $(\delta M,\delta Q_A)=(\neq 0,0)$ are in the stable region. What we found is that the unstable region is larger than we expected. However, it still does not have any volume in the tangent space because $\delta M=0$. We believe that the perturbation vectors $(\delta M,\delta Q_A)=(\delta M,1,a,b,-1-a-b)$ with small δM make the linear perturbation equations which involve all fields including the metric and are dynamically unstable. Therefore the unstable region has a volume in the tangent space.

It was argued that a unstable black string settles down to a new static black string solution which is not translationally invariant along the string and can be viewed as a

[2]We assume entropy is extremum at p, so that $\delta S=0$ in this tangent space.

local entropy maximum but not a global one [5]. If the final stage of the evolution is a local entropy maximum, we can not say that evolutions from different perturbatoins end up with the same final solution. In our case, we have 3 eigenvectors of Hessian with the same positive eigenvalues for all equal charge: $(a = 1, b = -1), (a = -1, b = 1)$ and $(a = -1, b = -1)$ in (9). Considering the sign of these vectors, there are six most unstable perturbation vectors in the tangent space. It would be very interesting to see that what will be the final solutions for these perturbations. We also would like to mention that evolutions in section 2 decrease entropy only at linear order. At late times, we need another description and it is more likely that entropy will increase in the end. It is also an open question that the evolutions from perturbations around the eigenvectors would result in the same final solutions in which the evolutions from the nearby eigenvectors result. We leave these questions for future work.

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